Impairments in Polarization-Multiplexed DWDM Channels due to Cross-Polarization Modulation

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***Abstract*—We derive the statistics of cross-polarization modulation-induced nonlinear crosstalk in polarization-multiplexed DWDM channels and an approximate relation between the degree of polarization and the associated noise terms.**

INTRODUCTION

Polarization division multiplexing (PDM) uses both polarization modes of a single-mode fiber to transmit two separate data streams (sub-channels) on a single optical carrier, effectively doubling the capacity of WDM channels. It has become a very attractive approach to transporting 100 Gigabit Ethernet on a single carrier [1]. Of the various linear and nonlinear effects which affect optical fiber transmission, cross-polarization modulation (XPolM) has a particularly detrimental influence on PDM and hence has attracted some interest recently [2]. The magnitude of XPolM in terms of the degree of polarization (DOP) reduction has previously been predicted by an analytical model [3], [4]. Herein, we try to characterize the PDM crosstalk caused by fast polarization modulation as a result of XPolM in terms of an equivalent additive noise process and estimate penalties depending on the magnitude of XPolM.

XPOLM IN (D)WDM SYSTEMS

XPolM describes the state of polarization (SOP) rotation of an optical signal in some arbitrary channel in a WDM system —which we will designate the *probe*— as a result of the nonlinear interaction between multiple channels [5]. The axis of this rotation is the sum of Stokes vectors of all other copropagating channels, and the angular velocity, or rotation rate, is given by the length of this Stokes sum scaled by the effective nonlinearity parameter, $8\gamma/9$ [2]. The lengths and polarizations (relative to the probe) of the Stokes vectors comprising this sum are subject to continuous random evolution as the signals propagate. Walk-off and pulse distortions affect their lengths, whereas frequency-dependent birefringence (PMD) alters their relative directions. In the model of [3], these random rotations result in a well-defined distribution of the polarization states of the probe after fiber propagation. This so-called Brownian distribution (so named for the random motion of the SOPs on the Poincaré sphere) is given in terms of the angle Θ between a particular SOP and the time-averaged SOP [6]. It is characterized by a single parameter, V, which is related to the more common DOP $\mathcal{D}$ by

$$\mathcal{D} = \exp\bigl(-V/2\bigr) \tag{1}$$

EFFECT OF POLARIZATION ROTATION ON PDM SYSTEMS

As a result of XPolM, the SOPs of the probe at the fiber output vary on a timescale of the bit period. Any PDM demultiplexer, optical or electrical, can only follow the average sub-channel SOPs on a much larger timescale. Each bit thus suffers a polarization misalignment during demultiplexing whose distribution is the Brownian mentioned above. As a result of this misalignment, the SOP of that bit is no longer orthogonal to the (average) SOP of the other sub-channel to which the demultiplexer is aligned. The loss of orthogonality consequently leads to crosstalk at the receiver. For illustration, we will label the sub-channels x and y, even though their actual polarizations are generally neither x nor y. If Θ is the misalignment angle in Stokes space, the (power) crosstalk P_{xy} from sub-channel x to sub-channel y derived from purely geometrical considerations in Jones space as

$$P_{xy} = P_x \frac{1-\cos\Theta}{2} \tag{2}$$

with the power P_x in sub-channel x of the probe before demultiplexing.

STATISTICAL ANALYSIS

The distribution of the crosstalk $\bar{P}_{xy} = P_{xy}/\bigl\langle P_x \bigr\rangle$ can be derived from the distribution of Θ via (2). It is well approximated by an exponential distribution with the parameter λ. To determine λ, we use the similarity of the Brownian and the Fisher distributions [6], [7] to obtain (for $\lambda \gtrsim 7$):

$$\lambda \approx \frac{2}{1-\mathcal{D}} \tag{3}$$

One can also show that the rotational symmetry of the (Brownian) SOP distribution in Stokes space corresponds to a uniformly distributed phase of the resulting demultiplex crosstalk field in Jones space. Since the phase is uniform, an exponential distribution of the crosstalk power (squared field magnitude) will correspond to a two-dimensional Gaussian distribution of the complex optical field. Fig. 1 shows the

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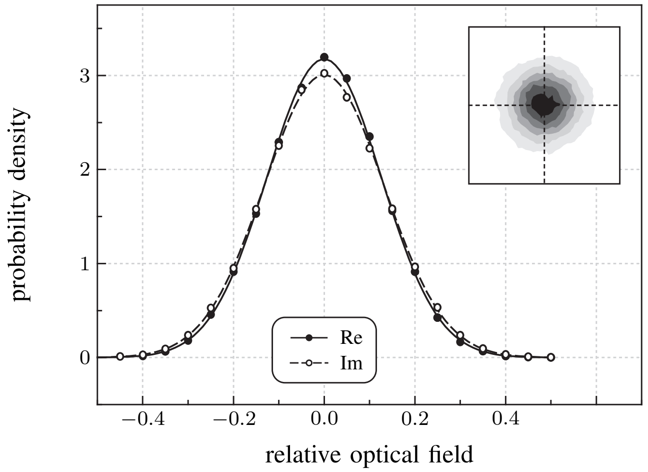

Fig. 1. Optical field in PDM sub-channel x due to the the crosstalk from sub-channel y; data obtained with vector NLSE simulations (symbols) and fit to Gaussian PDF (lines); $\mathcal{D} = 0.935$.

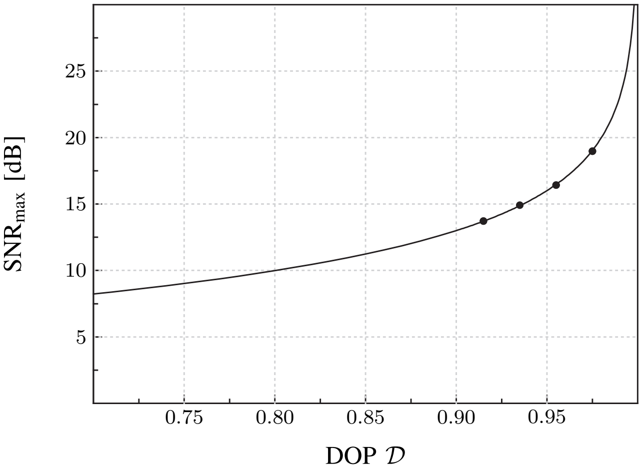

Fig. 2. Nonlinear crosstalk-limited SNR $\left\langle P_y \right\rangle / \left\langle P_{xy} \right\rangle$ as a function of the DOP $\mathcal{D}$ of PDM sub-channel x (assumed here to be a CW channel); markers denote numerical results.

crosstalk field obtained with numerical simulations, which indeed is Gaussian. The simulations used a CW probe to isolate the nonlinear polarization rotations due to XPolM as well as possible. The so predicted normalized crosstalk power for the system of Fig. 1 is $\left\langle \bar{P}_{xy} \right\rangle = (\lambda)^{-1} = 0.0325$. It is in very good agreement with the numerically obtained value of $\sigma_{\mathrm{re}}^2 + \sigma_{\mathrm{im}}^2 = 0.0156 + 0.0174 = 0.0330$, confirming our approach.

The stochastic process of nonlinear polarization rotations depends on the Stokes states of many WDM channels which are unknown in the signal path of the probe and cannot be compensated. The resulting crosstalk may therefore be regarded as demultiplexing *noise* in each PDM sub-channel, which is complex Gaussian but not spectrally white. Its average power is determined wholly by the DOP and power of the interfering channel via (2).

The XPolM-induced crosstalk noise sets an upper bound on the achievable SNR after the demultiplexer for a given DOP, independent of the signal bandwidth or format. This limit is plotted in Fig. 2. For an XPolM magnitude corresponding to DOP values as high as 0.95, the achievable SNR for any PDM sub-channel is limited by crosstalk alone to approx. 16 dB.

In a simplifying pump-probe approximation, $P_x > P_y$, the effect of P_{yx} on the SNR of channel y has been neglected in Fig. 2. To determine the accurate distorted field statistics after demultiplexing, one must additionally consider that the crosstalk terms $\bar{P}_{xy}$ and $\bar{P}_{yx}$ are highly correlated — Θ is the same for both polarizations at any fixed point in time. Thus, not only is the distortion originating in the orthogonal PDM sub-channel significant, but the magnitude of the signal itself is also reduced as some of its power is transferred away as crosstalk into the other direction. This will become relevant when calculating e.g. phase errors.

Amplifier ASE noise and the XPolM-induced crosstalk are uncorrelated and their average powers (variances) thus add linearly. To determine XPolM-related penalties on the required OSNR (ROSNR) in a communication link, one should thus calculate the mean ASE noise power corresponding to the ROSNR without XPolM, subtract the mean crosstalk power, and compute the new, higher, ROSNR. The exact penalty for some fixed crosstalk power will then depend on the modulation format and receiver filter characteristics, among others.

As an example, we may consider a 25 Gbaud DQPSK PDM system and assume an ROSNR value of about 17 dB (for a BER of 10^{-9} and 0.1 nm noise reference bandwidth). For simplicity, we assume a rectangular optical filter with 50 GHz (0.4 nm) bandwidth. Then, DOP values of 0.97/0.95/0.90 would correspond to ROSNR penalties of 1.0/1.6/4.3 dB.

SUMMARY AND CONCLUSIONS

We showed analytically and numerically how XPolM results in complex Gaussian demultiplex crosstalk noise in WDM-PDM systems, and that the noise power is determined by the DOP of the interfering PDM channel. It puts an upper limit on the obtainable SNR and can result in severe penalties on the required OSNR.

REFERENCES

[1] C. R. S. Fludger *et al.*, "Coherent equalization and POLMUX-RZ-DQPSK for robust 100-GE transmission," *J. Lightwave Technol.*, vol. 26, no. 1-4, pp. 64–72, Jan-Feb 2008.

[2] C. R. Menyuk and B. S. Marks, "Interaction of polarization mode dispersion and nonlinearity in optical fiber transmission systems," *J. Lightwave Technol.*, vol. 24, no. 7, pp. 2806–2826, July 2006.

[3] M. Winter *et al.*, "Analysis of cross-polarization modulation in dispersion-managed DWDM systems," in *Lasers and Electro-Optics Society, 2007. LEOS 2007. The 20th Annual Meeting of the IEEE*, 2007, paper WFF2.

[4] ——, "Cross-polarization modulation in DWDM systems," in *34th European Conference on Optical Communication (ECOC)*, 2008, accepted for publication.

[5] L. F. Mollenauer *et al.*, "Polarization scattering by soliton-soliton collisions," *Opt. Lett.*, vol. 20, no. 20, pp. 2060–2062, Oct 15 1995.

[6] P. H. Roberts and H. D. Ursell, "Random walk on a sphere and on a Riemannian manifold," *Philos. Trans. R. Soc. London, Ser. A*, vol. 252, no. 1012, pp. 317–356, 1960.

[7] N. I. Fisher *et al.*, *Statistical analysis of spherical data.* Cambridge, UK: Cambridge University Press, 1987.