

# Scaling of Nonlinear Threshold with Fiber Type and Channel Spacing in WDM Transmission Systems

Johannes Karl Fischer, Marcus Winter, Klaus Petermann

*Technische Universität Berlin, Fachgebiet Hochfrequenztechnik-Photonik, Einsteinufer 25,  
10587 Berlin, Germany  
j.k.fischer@ieee.de*

**Abstract:** It is shown that the impact of interchannel nonlinear effects in wavelength-division multiplexed fiber-optic transmission systems depends on channel spacing, the fiber's group-velocity dispersion and its attenuation coefficient, condensed into a single dimensionless parameter.

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## 1. Introduction

The ultimate channel capacity in fiber-optic wavelength-division multiplexed (WDM) transmission systems is limited by the inherent nonlinearity of the transmission medium silica fiber [1]. The nonlinear impairments originate from two distinct types of signal interaction. Interchannel nonlinearities arise from nonlinear interaction between different wavelength channels, while intrachannel nonlinearities describe the distortion which a single wavelength channel exerts on itself through fiber nonlinearity. Depending on system parameters such as bit rate, spectral efficiency and group-velocity dispersion of the transmission fiber the relative importance of inter- and intrachannel nonlinearities varies [2].

A parameter describing the impact of intrachannel nonlinearities depending on fiber type (attenuation and group-velocity dispersion) and bit rate was derived in previous work [3, 4]. However, this parameter is only universally valid for single-channel systems. As will be shown below, it is also valid for WDM systems, albeit for fixed spectral efficiency only. Since the relative importance of inter- and intrachannel nonlinearities and thus system performance varies strongly with spectral efficiency, it is desirable to find a normalization which is invariant towards spectral efficiency. In order to give system designers a simple metric, describing which combination of fiber type, bit rate and spectral efficiency is likely to give the best performance, an extended normalization is proposed in this contribution. Its validity is verified by means of numerical simulation for a wide variety of system configurations.

## 2. Theoretical Considerations

Starting from the nonlinear Schrödinger equation in the frequency domain and approximating its solution by a first order perturbation approach, the complex envelope of the electric field after transmission over a single fiber with subsequent dispersion compensation and reamplification can be written in the frequency domain as [5]

$$\tilde{A}(z, \omega) = \tilde{A}_0(\omega) + \delta_{NL}(z, \omega), \quad (1)$$

where  $\tilde{A}_0(\omega)$  is the complex field envelope at the input of the fiber and  $\delta_{NL}(z, \omega)$  is a small perturbation of the signal due to fiber nonlinearity. The perturbation depends on a double convolution of the input field envelope as [6]

$$\delta_{NL}(z, \omega) = j \iint \eta(\Delta\Omega) \tilde{A}_0(\omega_1) \tilde{A}_0^*(\omega_2) \tilde{A}_0(\omega - \omega_1 + \omega_2) d\omega_1 d\omega_2, \quad (2)$$

where  $\eta(\Delta\Omega)$  is the nonlinear transfer function and  $\Delta\Omega = (\omega - \omega_1)(\omega_1 - \omega_2)$ . For transmission over a single fiber with length  $L \gg 1/\alpha$  (with the fiber's attenuation coefficient  $\alpha$ ) the nonlinear transfer function can be approximated as

$$\eta(\Delta\Omega) = \int_0^L \gamma e^{(-\alpha - j\beta_2 \Delta\Omega)z} dz \approx \frac{\gamma}{\alpha} \frac{1}{1 - j\frac{\Delta\Omega}{\Omega_s}}, \quad (3)$$

where  $\Omega_s = -\alpha/\beta_2$  is its 3-dB bandwidth,  $\gamma$  the nonlinear coefficient and  $\beta_2$  the group-velocity dispersion of the fiber. Relating the 3-dB bandwidth of the nonlinear transfer function to a measure for the spectral width of a single channel (e. g. the bit rate  $B$ ) yields a normalized dispersion  $C_1 = -\frac{\beta_2}{\alpha} B^2$  [4]. This dimensionless parameter is directly

proportional to the number of nonlinearly interacting bits over the effective length ( $L_{eff} \approx 1/\alpha$ ) of a fiber and thus describes the strength of intrachannel nonlinearities [7]. Eq. (3) is also closely related to analysis of interchannel nonlinear effects, where  $|\eta(\Delta\Omega)|$  describes the efficiency of four-wave mixing (FWM) and  $\beta_2\Delta\Omega$  the phase mismatch of the interacting fields [8]. Therefore, relating the 3-dB bandwidth of the nonlinear transfer function to the separation between wavelength channels yields a parameter characterizing the strength of FWM in a given system configuration. This parameter also takes the form of a dimensionless normalized dispersion and can be written as (with  $\Delta\Omega \propto \Delta f_{ch}^2$ )

$$C_2 = -\frac{\beta_2}{\alpha} \Delta f_{ch}^2 = -\frac{\beta_2}{\alpha} \frac{B^2}{S^2}, \quad (4)$$

where  $\Delta f_{ch}$  is the channel spacing between wavelength channels in units Hz and  $S$  is the spectral efficiency.

### 3. Simulation Setup

In order to verify the proposed normalization, transmission of five WDM channels over a single span is numerically simulated with the commercially available software tool VPItransmissionMaker. A schematic of the simulation setup is shown in Fig. 1(a). The transmitters (Tx) generate conventional chirp-free non-return-to-zero on-off-keying (NRZ-OOK) with a continuous-wave laser followed by a Mach-Zehnder modulator in push-pull configuration. The transmitted data sequence is a de Bruijn binary sequence of length  $2^{10}$ , with different delays for each wavelength channel to decorrelate the sequences. All channels are copolarized and are assumed to be phase-locked. Considered bit rates are 10, 20 and 40 Gb/s with spectral efficiencies of 0.1, 0.2 and 0.4 b/s/Hz. The considered spectral efficiency is restricted to  $S \leq 0.4$  b/s/Hz in order to avoid coherent WDM crosstalk, which could cause misleading results [9]. Multiplexer and demultiplexer filters are modeled as second order Gaussian with a 3-dB bandwidth of twice the bit rate. Optical spectra after the multiplexer filters are shown in Fig. 1(b)-(d). The dispersion precompensation is optimized for every simulation point. The single span consists of 80 km single-mode fiber (SMF) with attenuation coefficient  $\alpha = 0.2$  dB/km, nonlinear coefficient  $\gamma = 1.31$  W<sup>-1</sup>km<sup>-1</sup> and varying group-velocity dispersion  $\beta_2$ . Second-order group-velocity dispersion is not considered in this analysis. The SMF is followed by a dispersion-compensating fiber (DCF), which is here considered as a linear transmission medium due its to low input power. The DCF is dimensioned for zero net residual dispersion. At the receiver (Rx), the photodiode is modeled as an ideal square-law device, followed by a 5<sup>th</sup>-order Bessel low-pass filter with a cut-off frequency of  $0.7B$ . The required optical signal-to-noise ratio (OSNR) in a bandwidth of 0.1 nm for a BER of  $10^{-9}$  is estimated using a method presented in [10]. A practical criterion for the impact of fiber nonlinearity on a system is the nonlinear threshold (NLT). It is defined as the launch power resulting in a 1 dB penalty in required OSNR.

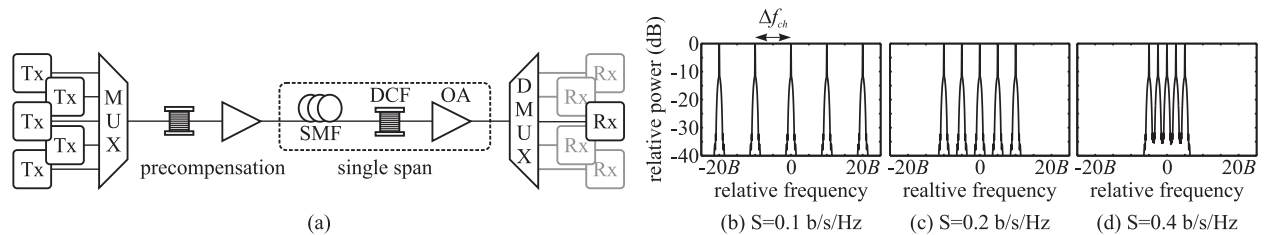


Fig. 1. Schematic of the simulation setup (a) and optical spectra of NRZ-OOK WDM signals consisting of five wavelength channels with bit rate per channel  $B$  and spectral efficiency of (b) 0.1 b/s/Hz, (c) 0.2 b/s/Hz and (d) 0.4 b/s/Hz.

### 4. Results and Discussion

Fig. 2(a) plots the NLT as a function of normalized dispersion  $C_1$ , which was proposed for characterization of single-channel systems in [4]. While the single-channel curve is invariant to changes of fiber type and bit rate, there are three distinct curves for different spectral efficiencies in case of WDM transmission. Only for large bit rate ( $B \geq 40$  Gb/s) and dispersion ( $D \geq 8$  ps/(nm·km)), i. e. for  $C_1 \geq 0.3$ , the three curves converge and the NLT becomes independent of spectral efficiency. In this regime intrachannel nonlinearities are limiting the NLT. Nevertheless, for fixed spectral efficiency the curves are still invariant to bit rate and fiber type, regardless of the predominant type of nonlinearity.

When plotting the NLT against the normalized dispersion  $C_2$  in Eq. (4) the curves become invariant to spectral efficiency for system configurations where interchannel nonlinearities limit the NLT, i. e.  $C_2 < 1$  in Fig. 2(b). However, the proposed normalization does not capture the impact of intrachannel nonlinearities. Consequently, the curves

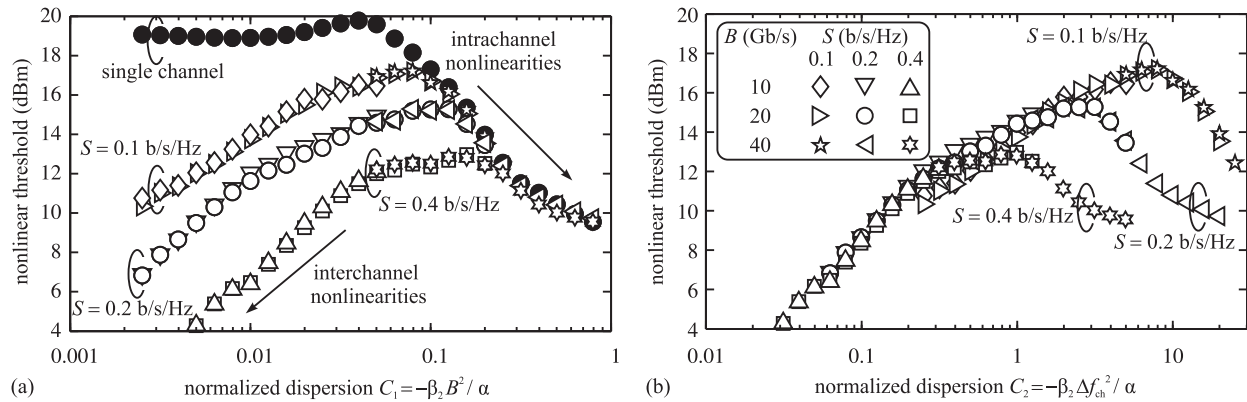


Fig. 2. Nonlinear threshold in dBm at optimized precompensation as a function of (a) normalized dispersion  $C_1$  derived in [4] and (b) normalized dispersion  $C_2$  as defined in Eq. (4) for varying spectral efficiency  $S$ . The legend for both graphs is shown in the figure on the right hand side.

for systems impaired mostly by intrachannel nonlinearities diverge. Comparison of Fig. 2(a) and (b) reveals a dualism. While the normalization  $C_1$  universally describes the scaling of nonlinear threshold in systems predominantly impaired by intrachannel nonlinearities, the same is true for normalization  $C_2$  in systems impaired by interchannel nonlinearities. Both normalizations fail when the respective other type of nonlinearity limits the NLT. Therefore it can be concluded that no single parameter exists, which describes the scaling of nonlinear threshold for arbitrary bit rate, fiber type and spectral efficiency. However, in recent years the possibility to electronically compensate for intrachannel nonlinearities has emerged [11]. The remaining nonlinear impairments are caused by interchannel nonlinearities. It can be expected that normalization  $C_2$  is universally valid in these systems. Although only a single span system is considered here, the results can be adapted to multi-span systems. Moreover, single-span configurations giving maximum nonlinear threshold can be used to estimate optimum multi-span system configurations [12].

## 5. Conclusion

The strength of nonlinear impairments in WDM transmission systems induced by interchannel nonlinearities can be described by a simple dimensionless parameter, depending on group-velocity dispersion and attenuation of the optical fiber as well as channel spacing. This parameter could prove especially useful for the design of future transmission systems, where intrachannel nonlinearities can be compensated electronically (e. g. systems considered in [1]) and interchannel nonlinearities pose the ultimate limit to channel capacity.

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