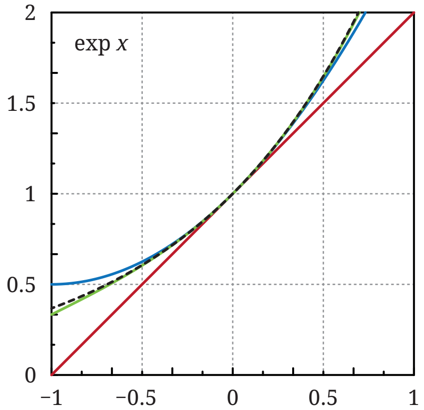
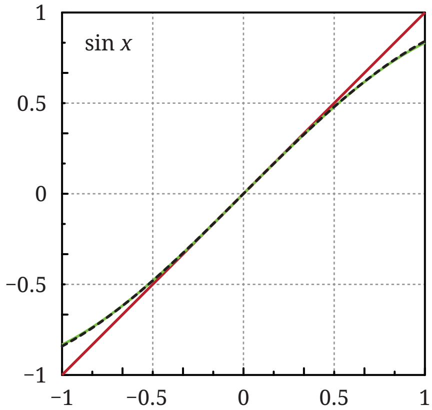
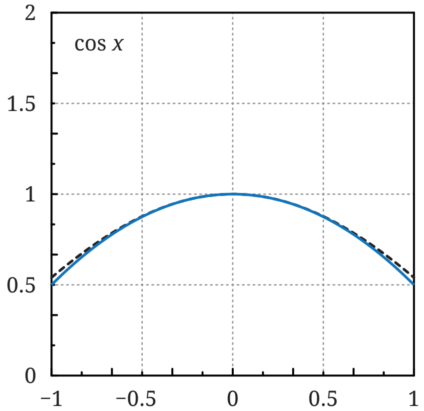
Series Expansions Cheat Sheet



$$\exp x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{6} + \ldots$$

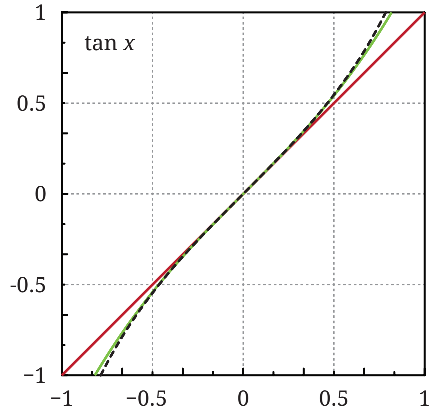


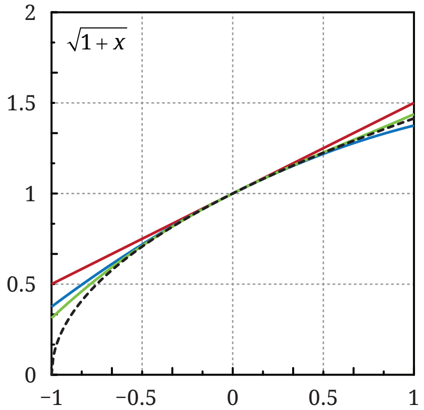


$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2} + \ldots$$

$$\tan x = \sum_{n=0}^{\infty} \frac{A_{2n+1}}{(2n+1)!} x^{2n+1} = x + \frac{x^3}{3} + \ldots$$

Euler zigzag numbers A_n



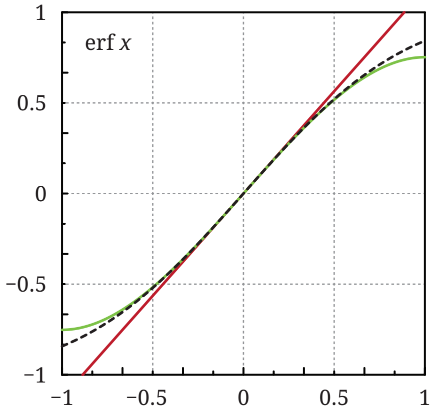


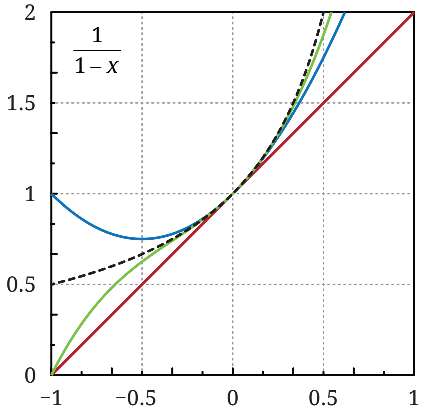
$$\sqrt{1+x} = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{(1-2n)(n!)^2 4^n} x^n$$

$$= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \ldots$$

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)n!} x^{2n+1}$$

$$= \frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{3} + \ldots \right)$$



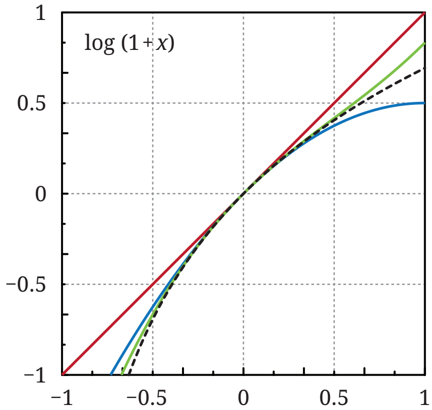


$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots$$

geometric series

$$\log(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$$

$$\log_b x = \log x \cdot \log_b e$$



Taylor series $f(x) = \sum_{n=0}^{\infty} \frac{(x-a)^n}{n!} \frac{d^n}{dx^n} f(a)$ (Maclaurin series when $a = 0$)

general power series $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$ corresponds to Taylor series for $c_n = \frac{1}{n!} \frac{d^n}{dx^n} f(a)$